# Strongly Coupled Charged Scalar In B and T Decays

#### PAWEŁ KRAWCZYK

Institute for Theoretical Physics,
Warsaw University

#### STEFAN POKORSKI\*

Fermi National Accelerator Laboratory P.O. Box 500, Batavia, IL 60510

#### Abstract

Limits on charged scalar Yukawa couplings from  $\tau$  and B decays are discussed. They (and other existing limits) are consistent with "strong" couplings (0(1)) for the third generation) even if the lightest scalar mass is in the range  $20~{\rm GeV}/c^2 \lesssim M_\phi \lesssim 100~{\rm GeV}/c^2$  but saturated in this case.  $BR(B \to \tau \nu X)$  and (for  $M_\phi > m_t$ ) BR  $(T \to \tau \nu X)$  may be then as large as 30% and 70%, respectively. Both for  $M_\phi < m_t$  and  $M_\phi > m_t$  the potentially possible top quark signature in  $p\bar{p}$  collisions is  $(\tau + 2jets)$  final state. The upper limit for  $\tau \to \nu \eta \pi$  is 0(.003%).

<sup>\*</sup>On leave of absence from the Institute for Theoretical Physics, Warsaw University, Warsaw, Poland.

#### I. Introduction

In most "beyond the standard" models several Higgs doublets are present and consequently the weak forces are mediated, in addition to the intermediate vector bosons, by charged scalar particles. Models with two and three scalar doublets have been explicitly studied and some constraints on the quark Yukawa couplings have been derived from the  $K^0 - \overline{K}^0$ ,  $D^0 - \overline{D}^0$  and  $B^0 - \overline{B}^0$  mixing<sup>[1,2]</sup> and from the CP violation.<sup>[3]</sup> One appealing possibility is that in three (or more) doublet models the hierarchy of the vacuum expectation values is such that the Yukawa couplings are of the same order for the members of a given heavy generation and O(1) for the third generation (we shall call such couplings "strong").

On phenomenological side it was repeatedly pointed out that the data in the light lepton and quark sector still admit relatively large deviations from the V-A structure of the charged currents.

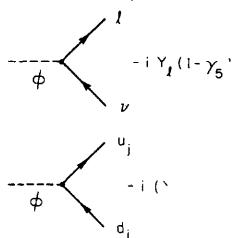
A recent thorough analysis<sup>[4]</sup> of the high precision data (including polarization measurements) on muon decay, inverse muon decay,  $\pi l2$  decays and nuclear Gamov-Teller transitions shows that in those reactions the effective scalar coupling might be of the order of ten percent of  $G_F$ , provided it is proportional to the lepton mass.

In this paper we analyse the limits on the charged scalar couplings from the  $\tau$  and B decays. In three (or more) doublet models with "natural" absence at the tree level of flavour changing neutral currents there are two independent sets of Yukawa couplings, driving the up and down quark masses, respectively. Our bounds are consistent with both being O(1) for the third generation and they are in fact saturated by such strong couplings if the lightest scalar mass is in the range  $20 \text{ GeV} \leq M_{\phi} \leq 100 \text{ GeV}$ . In this case the branching ratios  $BR(B \to \tau \nu X)$  and (for  $M_{\phi} > m_t$ )  $BR(T \to \tau \nu X)$  may be as large as 30% and 70%, respectively. Thus, those channels are very restrictive for potential scalar exchange.

In particular, in view of the potential possibility for the branching ratio for  $B \to \tau \nu X$  to be so large, it is of clear interest to use the available data to place better limits on this decay mode. This should be possible by analyzing the momentum spectrum of the electrons in the decay  $B \to e X$ . The shape of the spectrum would be sensitive to a large contamination of  $B \to \tau \nu X$  with the subsequent decay  $\tau \to e \nu \nu$  (we thank U. Baur for discussion on this point).

II. 
$$\tau - decays$$

Our notation (for effective couplings) is as follows:



We also define the effective scalar couplings G, e.g.

$$G_{ij,l}^{L} = \frac{Y_{ij}^{L}Y_{l}}{M_{\Phi}^{2}}, \quad G_{l,l} = \frac{Y_{l}Y_{l}}{M_{\Phi}^{2}} \text{ etc.}$$
 (1)

and the ratios H, e.g.

$$G_{ij,l}^L/G_F \equiv H_{ij,l}^L$$
 
$$G_{l,l}/G_F \equiv H_{l,l} \qquad (2)$$

As it has already been mentioned, from the  $e - \mu$  sector one gets the limit<sup>[4]</sup>

$$H_{\epsilon,\mu} < 0.22 \tag{3}$$

provided that

$$Y_{\epsilon} = \frac{m_{\epsilon}}{v_l}, Y_{\mu} = \frac{m_{\mu}}{v_l} \tag{4}$$

With scalar exchange included, the rate for the decay  $\tau \to \mu\nu\overline{\nu}$  reads:

$$\Gamma_{\tau \to \mu \nu \bar{\nu}} = \frac{G_F^2 m_\tau^5}{192 \pi^3} \left( 1 + \frac{1}{32} H_{\tau,\mu}^2 \right) \left[ 1 - \frac{m_\mu}{m_\tau} \frac{H_{\tau,\mu}}{\sqrt{2} \left( 1 + \frac{1}{32} H_{\tau,\mu}^2 \right)} - 8 \frac{m_\mu^2}{m_\tau^2} + 0 \left( \frac{m_\mu^4}{m_\tau^4} \right) \right]$$
(5)

Analogous formula holds for  $\tau \to e\nu\overline{\nu}$  but due to eq.(4) the scalar exchange contribution is negligible as compared to the contribution to  $\tau \to \mu\nu\overline{\nu}$ . Now, the ratio

$$R \equiv \frac{\Gamma(\tau \to \mu \nu \overline{\nu}) - \Gamma(\tau \to e \nu \overline{\nu})}{\Gamma(\tau \to e \nu \overline{\nu})} = (0.016 \pm 0.037) \tag{6}$$

can be used to place the limit on  $H_{\tau,\mu}$ . Using eq.(5) we have

$$R = -\frac{H_{\tau,\mu}}{\sqrt{2}} \frac{m_{\mu}}{m_{\tau}} + \frac{H_{\tau,\mu}^2}{32} - 8 \frac{m_{\mu}^2}{m_{\tau}^2} \left( 1 + \frac{H_{\tau,\mu}^2}{32} \right) \tag{7}$$

and, from (6),

$$\left|H_{\tau,\mu}\right|\lesssim 2.5\tag{8}$$

Let us now study the scalar exchange contribution to the hadronic  $\tau$  decay modes. The rate for  $\tau \to \pi^- \nu$  is measured with good accuracy. The full amplitude reads

$$M = \frac{G_F}{\sqrt{2}} \left\{ f_{\pi} k^{\mu} \cos \Theta_c \overline{\nu}_{\tau} \gamma_{\mu} (1 - \gamma_5) \tau \varphi_{\pi}^{\bullet} + \frac{Y_{\tau}}{2\sqrt{2}G_F M_{\phi}^2} \left( Y_{ud}^R - Y_{ud}^L \right) < \pi^- |\overline{d}\gamma^5 u| 0 > \overline{\nu}_{\tau} (1 + \gamma_5) \tau \right\}$$
(9)

where  $k^{\mu}$  is the pion four-momentum,  $f_{\pi}$  and  $\varphi_{\pi}$  its decay constant and wave function, respectively, and  $\Theta_{\varepsilon}$  is the Cabibbo angle. One gets

$$\Gamma(\tau \to \pi \nu) = \frac{G_F^2 f_\pi^2 \cos^2 \Theta_c}{16\pi} m_\tau^3 \left( 1 - \frac{m_\pi^2}{m_\tau^2} \right)^2 \\ \left| 1 + i \frac{(H_{ud,\tau}^R - H_{ud,\tau}^L) < \pi^- |\bar{d}\gamma^5 u| 0 > \varphi_\pi}{2\sqrt{2} m_\tau f_\pi \cos \Theta_c} \right|^2$$
(10)

In spite of the apparent correction the prediction in eq.(10) cannot be distinguished from the one in the standard model provided  $Y_{\tau}/Y_{\mu} = m_{\tau}/m_{\mu}$ . Indeed, the pion decay constant  $f_{\pi}$  is measured in the decay  $\pi \to \mu \bar{\nu}$  whose decay rate is in this case subject to the same (numerically) correction. Thus, presence of the scalar contribution leads to a rescaling of  $f_{\pi}$  but does not affect the standard model prediction for  $\tau \to \pi \nu$  and no bound can be obtained from this process. Analogous result holds for the decay  $\tau \to K \nu$  and the kaon decay constant  $f_K$ .

We are nevertheless able to get certain constraint from the fact that the effective  $\tilde{f}_{\pi}$  and  $\tilde{f}_{k}$  satisfy  $\tilde{f}_{\pi}/\tilde{f}_{K} \approx 1/1.27$ . Thus

$$1.27 f_{\pi} \left| 1 + i \frac{\left( H_{ud,\tau}^{R} - H_{ud,\tau}^{L} \right) < \pi |\overline{d}\gamma^{5}u| 0 > \varphi_{\pi}}{2\sqrt{2}m_{\tau}f\pi \cos\Theta_{c}} \right|$$

$$= f_{K} \left| 1 + i \frac{\left( H_{us,\tau}^{R} - H_{us,\tau}^{L} \right) < K|\overline{s}\gamma^{5}u| 0 > \varphi_{k}}{2\sqrt{2}m_{\tau}f_{K}\sin\Theta_{c}} \right|^{2}$$

$$(11)$$

and the pseudoscalar current matrix elements can be evaluated from PCAC<sup>[6]</sup>:

$$|\langle \pi | \overline{d} \gamma^5 u | 0 \rangle| = \frac{\sqrt{2} f_{\pi} m_{\pi}^2}{m_u + m_d}$$

$$|\langle K | \overline{s} \gamma^5 u | 0 \rangle| = \frac{\sqrt{2} f_K m_K^2}{m_s + m_u}$$
(12)

Since we expect  $f_{\pi} \approx f_K$ , under more specific assumption about the scalar sector eq. (11) and eq. (12) can provide approximate constraint on the couplings  $H_{su,\tau}$  and  $H_{du,\tau}$ . This we explore in the context of the three doublet model.

### III. B-decays

In the standard model we have

$$\Gamma(B \to e\nu X) = \frac{G_F^2 m_b^5}{192\pi^3} \left[ C_{ce} |V_{bc}|^2 + C_{ue} |V_{bu}|^2 \right]$$
 (13)

where the numerical factors  $C_{ce}$  and  $C_{ue}$  contain phase space effects and QCD corrections for the transitions  $b \rightarrow ce\nu$  and  $b \rightarrow ue\nu$ , respectively. Eq. (13), rewritten in the form

$$\Gamma(B \to e\nu X) = \frac{G_F^2 m_b^5}{192\pi^3} C_{ce} |V_{bc}|^2 [1+R] \tag{14}$$

where

$$R = \frac{C_{ue}|V_{bu}|^2}{C_{ce}|V_{bc}|^2} \tag{15}$$

is used to determine  $|V_{bc}|^2$  given the experimental data for  $BR(B \to e\nu X)$ , the B meson life time, the ratio R (from the momentum spectrum of leptons) and the "theoretical" value of  $C_{ce}$ ,  $C_{ce} \approx 0.4^{[7]}$  (with 5% error). Since we focus on scalar

couplings proportional to the lepton mass the result (13) does not change even with the scalar exchange present. Let us now consider the channel  $b \to c\tau\nu$  (and similarly  $b \to c\bar{c}s$  but, as we shall see in the next section, in multi-doublet models the scalar contribution to the latter is strongly suppressed): <sup>2</sup>

$$\Gamma(B \to X \tau \nu) = \frac{G_F^2 m_b^5}{192 \pi^3} \left[ C_{e\tau} |V_{be}|^2 + C_{u\tau} |V_{bu}|^2 + \Delta_{\tau} |V_{be}|^2 \right]$$
(16)

where

$$\Delta_{\tau} = \frac{1}{32} \left( \left| H_{bc,\tau}^{L} \right|^{2} + \left| H_{bc,\tau}^{R} \right|^{2} \right) C_{c\tau} \frac{1}{|V_{bc}|^{2}}$$

$$C_{c\tau} = 0.06 \tag{17}$$

To our knowledge there is no experimental upper limit for BR  $(B \to X \tau \nu)$  but we can place a limit on  $\Delta$  from its corresponding reflection on the total width and on the predicted BR  $(B \to e\nu X)$ . The total width is:

$$\Gamma(B \to X) = \frac{G_F^2 m_b^5}{192\pi^3} \left[ C_c |V_{bc}|^2 + C_u |V_{bu}|^2 + (\Delta_\tau + \Delta_c) |V_{bc}|^2 \right]$$
(18)

where  $\Delta_c$  describes the scalar contribution to  $b \to c \bar{c} s$ . The coefficients  $C_c$  and  $C_u$  are subject to various uncertainties mainly due to the choice of the quark mass values and to the strong interaction effects.<sup>[8]</sup> A fair choice<sup>[9]</sup>  $C_c = 2.75$  together with experimental branching ratio  $BR(B \to e\nu X) = (12.3 \pm 0.9)\%$  leaves some room for scalar contribution. Using eq.(13), eq.(18),  $C_{ce} \approx 0.4$  and setting  $R = C_{ue}|V_{bu}|^2/C_{ce}|V_{bc}|^2 = 0$  (experimentally R < 0.03) we get

$$\Delta_{\tau} + \Delta_{c} \lesssim 0.7 - 0.8 \tag{19}$$

#### IV. Multi-doublet Models

As it is well known, in such models the flavour changing neutral currents are "naturally" (irrespectively of the values of the Yukawa couplings) eliminated at the tree level if only one scalar doublet couples to the up quarks and one other

<sup>&</sup>lt;sup>2</sup>In the first approximation we neglect the interference between V-A and scalar amplitudes. This is justified a posteriori by the value of the upper bound on  $\Delta$  as compared to  $C_{c\tau}$ 

scalar is coupled to the down quarks.<sup>[10]3</sup> In addition, for  $Y_l = m_l/v$ , only one scalar must couple to leptons (one of the two or one other). We do not consider here the possibility that the tree level flavour changing neutral currents are avoided by an "ad hoc" adjustment of large Yukawa couplings.

In the mass eigenstate basis the charged scalar couplings to fermions read:

$$L_{Y}^{+} = 2^{3/4}G_{F}^{1/2}\overline{U}\left[M_{U}K\sum_{i}Y_{i}\Phi_{i}^{+}P_{L} - KM_{D}\sum_{i}X_{i}\Phi_{i}^{+}P_{R}\right]D$$

$$- \nu M_{L}P_{R}L\sum_{i}X_{i}\Phi_{i}^{+} + h.c.$$
(20)

where K is the Kobayashi-Maskawa matrix and  $M_U$ ,  $M_D$  and  $M_L$  are mass matrices for the up, down quarks and leptons, respectively. The explicit expressions for the constants  $X_i$  and  $Y_i$  in terms of the vacuum expectation values are well known in the two-and three-doublet models.<sup>[11]</sup>

In the two doublet model X = 1/Y. In the three doublet model there is no such constraint and both couplings may approach the perturbativity limit

$$2^{3/4}G_F^{1/2}|Y_i|m_t \sim 2^{3/4}G_F^{1/2}|X_i|m_b \sim 0(\sqrt{4\pi})$$
 (21)

These limits are consistent with the constraints derived from the renormalization group approach, eq. (18) in [12], which give  $|Y_i| \leq 5$ ,  $|X_i| \leq 50$ .

We have assumed leptons and down quarks to get masses from the same scalar doublet since, as we shall see, in both two-and three-doublet models this gives the strongest coupling of leptons to scalars.

In several previous papers the limits on  $Y_i's$  have been derived from the  $K^0 - \overline{K}^0$  and  $B^0 - \overline{B}^0$  mixing [1,2] and from the CP violation in the kaon system. Roughly speaking, for light scalars  $(M_{\phi} \approx (20-100)GeV)$ , those limits coincide with the perturbativity limit (21) (one gets  $|Y| \leq 2\sqrt{\frac{M_{\phi}}{m_i}}$ ). As long as  $X_i \leq Y_i$  the effective scalar couplings are then some orders of magnitude below the experimental limits (3), (8) and (19), even for  $M_{\phi} \approx 20GeV$ . At the tree level, only t decays are sensitive to the Y coupling. However, it is perfectly possible that  $X_i \gg Y_i$  (e.g. both Yukawa

<sup>&</sup>lt;sup>3</sup>There is also a less general possibility to couple all quarks to only one scalar. The discussion can be easily restricted to this case and also the bounds on the Yukawa couplings become more restrictive.

couplings are equally important for the third generation) and then  $X_i$  couplings dominate in all but t decay sectors. <sup>4</sup>

Neglecting in the first approximation all but the lightest Higgs exchange contribution to the effective coupling we get from (20)

$$H_{su,\tau}^R/H_{du,\tau}^R = tg\Theta_c \frac{m_s}{m_d} \approx \sqrt{\frac{m_s}{m_d}}$$
 (22)

Eq.(11) can now be used to place limits on  $H^R$ s if we make a plausible assumption that  $1 < 1.27 f_{\pi}/f_{K} < 1.27$ . One gets

$$H_{du,\tau}^R \lesssim 0.3, \quad H_{gu,\tau}^R \lesssim 1.3 \tag{23}$$

When expressed in terms of X the limits (8) and (23) are similar and give

$$|X| \lesssim 2[M_{\phi}/1 \text{ GeV}] \tag{24}$$

whereas the limit (3) is somewhat weaker.

It turns out that the strongest limit for X is provided by B decays, eq. (19). We get

$$|X| \lesssim 0.9[M_{\phi}/1 \text{ GeV}] \tag{25}$$

The difference between the bounds (25) and (24) is important since cross sections are proportional to  $X^4$ . For instance the bound (25) makes the scalar contribution to  $\tau \to \mu\nu\nu$  smaller than 0.4%. On the other hand, if (24) was saturated the  $B \to \tau\nu X$  decay would saturated the total B width, in contradiction to experimental evidence for  $B \to e\nu X$ ,  $B \to \mu\nu X$  and simple quark counting. Since eq.(25) is the strongest limit presently available we can use eqs. (6),(18) and (19) to estimate the upper limit for  $BR(B \to \tau\nu X)$ :

$$BR \lesssim 30\%$$
 , (26)

to be compared to the standard model result  $\sim 2.5\%$ .

For the top quark decay  $t \to b\tau\nu$  we have

$$\Gamma(t \to b\tau\nu) = \frac{G_F^2 m_t^2}{192\pi^3} \left[ 1 + \frac{(m_t m_\tau)^2}{4} \left[ (YX)^2 + (\frac{m_b}{m_t})^2 X^4 \right] \right] \frac{1}{M_d^4}$$
 (27)

<sup>&</sup>lt;sup>4</sup>The following discussion, including eq.(24) and (25) holds for both two-and three-doublet models.

Taking the upper limit  $(X/M_{\phi})^4 = 0.7 \text{ GeV}^{-4}$  and  $Y = \frac{1}{2} \frac{m_t}{m_t} X$ , 5  $m_t \approx 50 \text{ GeV}$ , we get the bound (if  $M_{\phi} > m_t$ )<sup>6</sup>

$$BR(t \to \tau \nu X) \lesssim 70\% \tag{28}$$

and also the top hadron life-time about 2 times shorter than in the standard model. The branching ratios for the semi-leptonic decays into electrons and muons would be correspondingly smaller than in the standard model.

Given the bound (25) we can estimate the contribution of scalar exchange to channels strongly suppressed in the standard model, like e.g.G = -1, P = +1 decay mode  $\tau \to \nu \pi \eta$ . This is of interest in the context of the "missing mode" puzzle in  $\tau$  decays.<sup>[14]</sup> The order of magnitude estimate is given by the inclusive rate  $\tau \to \phi \nu \to q \bar{q} \nu$ . We get .003% as the upper limit for  $\tau \to \nu du$  and 0.06% for  $\tau \to \nu su$ .

Finally, let us comment on the  $K^0-\overline{K}^0$ ,  $B-\overline{B}^0$  and  $D^0-\overline{D}^0$  mixing. It is not difficult to see that the first two mainly constrain the Y couplings and the last one is sensitive to the X-couplings.<sup>[1,2]</sup> Our calculation with both terms present (Lagrangian (20)) shows that the  $K^0-\overline{K}^0$  and  $B^0-\overline{B}^0$  mixing is consistent with the derived bounds.<sup>[15]</sup> For  $D^0-\overline{D}^0$  mixing we get  $\Delta m_D\approx 0(10^{-16}~{\rm GeV})$  (for  $M_\phi=50~{\rm GeV}$  and using (25)).

## V. Conclusions

Strongly coupled scalars, with couplings 0(1) for the third generation, have several theoretically attractive features, as advocated for instance in ref.[3]. The bounds on the Yukawa couplings derived in this paper from  $\tau$  and B decays are consistent with "strong" couplings. (The specific set of parameters proposed in ref. [3] in a model for the CP violation seems, however, to be ruled out by our bounds). If the mass of the lightest scalar is in the range 20 GeV  $\lesssim M_{\phi} \lesssim 100$  GeV those bounds are saturated by such strong couplings. Thus, strongly coupled scalars

<sup>&</sup>lt;sup>5</sup>For  $M_{\phi} \approx m_t$  this agrees with  $|Y| < 2\sqrt{\frac{M_{\Phi}}{m_t}}$ 

<sup>&</sup>lt;sup>6</sup>If  $M_{\phi} < m_t$  the top quark total decay width is within very good approximation given by  $t \to b\phi$  and the BR for other decay modes are very small.

in that mass range, if they exist, should be soon observed directly and indirectly.

The branching ratios for  $B \to \tau \nu X$  and if  $(M_{\phi} > m_t)$  for  $T \to \tau \nu X$  are most promising and most restrictive for potential scalar exchange. With the present limits saturated they may be as large as 30% and 70%, respectively. For  $M_{\phi} > m_t$  the top quark decay signature would then be  $t \to \tau +$  jet and for  $m_t > M_{\phi}$  the decay  $t \to b\phi^+ \to \tau +$  jet could be the dominant one. In both cases the top quark signature in  $p\bar{p}$  collisions could potentially be the final state  $\tau + 2$  jets(the second jet is expected from  $W \to tb$  decay).

Large contribution of  $B \to \tau \nu X$  with the subsequent decay  $\tau \to e\nu\nu$  would reveal itself in the shape of the momentum spectrum of electrons in  $B \to eX$ . Such an analysis is perhaps possible even with the presently available data.

All the limits have been derived under the assumption of "natural" absence of flavour changing neutral currents but they do not depend on any further details of the Higgs sector.

#### Acknowledgments

We are grateful to S. Nandi and H. Reno for useful conversations. This work is partially supported by the Polish Ministry of Science and Higher Education under the grant CPBP 01.03.

## References

- [1] L. F. Abbott, P. Sikivie and M. B. Wise, Phys. Rev. **D21** (1980), 1393
- [2] G. G. Athanasiu, P. J. Franzini and F. J. Gilman, Phys. Rev. **D32** (1985), 3010
- [3] G. C. Branco, A. J. Buras and J. M. Gérard, Nucl. Phys. B259(1985), 306
- [4] K. Mursula, M. Roos and F. Scheck, Nucl. Phys. B219(1983), 321 K. Mursula and F. Scheck, Nucl. Phys. B253(1985), 189
- [5] For a recent review see M.G.D. Gilchriese, Rapportear talk at the Berkeley Conference, 1986
- [6] See, for instance, S. Pokorski, Gauge Field Theories, Cambridge University Press, 1987; p.262

- [7] J. Lee-Franzini, Invited Talk at the Europhysics Topical Conference "Flavour Mixing in Weak Interaction" Erice 1984
- [8] M. K. Gaillard and L. Maiani, Proceedings of the 1979 Cargese Summer Institute on Quarks and Leptons, edited by M. Levy et.al., Plenum New York, 1979. J. L. Cortes, X. Y. Pham and A. Tounsi, Phys. Rev. D25 (1982), 188; G. L. Fogli, Phys. Rev. D28(1983) 1153; N. Cabibbo and L. Maiani, Phys. Lett 79 B (1978), 109; G. Altarelli, G. Curci, G. Martinelli, S. Petrarca, Phys. Lett 99 B (1981), 141
- [9] M. K. Gaillard and L. Maiani, ref. [8]; J. L. Cortes et al., ref. [8]
- [10] S. Glashow and S. Weinberg, Phys. Rev. D15(1977), 1958
- [11] C. H. Albright, J. Smith and S. N. Tye, Phys. Rev **D21**(1980), 711
- [12] J. Bagger, S. Dimopoulos and E. Masso, Phys. Lett. 156 B(1985), 357
- [13] G. G. Athanasiu and F. J. Gilman, Phys. Lett. 153 B(1985), 274
- [14] F. J. Gilman and S. H. Rhie, Phys. Rev. D31 (1985), 1066; F. J. Gilman, SLAC-PUB-4265, March 1987
- [15] P. Krawczyk and S. Pokorski, to be published.